

Brief Report on the Current Situation Surrounding Inter-universal Teichmüller Theory (IUT)

Shinichi Mochizuki

August 2023

Despite

- the **publication in 2021 of the four main papers** [\[IUTchI\]](#), [\[IUTchII\]](#), [\[IUTchIII\]](#), [\[IUTchIV\]](#) on inter-universal Teichmüller theory (IUT) in a special volume of Publ. RIMS edited by a special editorial board consisting of such internationally renowned mathematicians as M. Kashiwara, A. Tamagawa, S. Mukai, H. Nakajima, and T. Mochizuki;
- the **subsequent publication of the sequel** [\[ExpEst\]](#) on explicit estimates in IUT in the Kodai Math. J. in 2022;
- the release of **detailed surveys** (cf. [\[Alien\]](#), [\[EssLgc\]](#)) concerning IUT;
- numerous **workshops/talks/videos** (cf. [WS3](#), [WS4](#)) on IUT conducted/produced during the period 2012 – 2023;
- a pronounced shift in recent years in the focus of activities of many researchers involved with IUT from activities devoted to disseminating the original version of IUT to activities centered around the development of various **new versions/applications of IUT**, many of which are closely related to the **Section Conjecture** in anabelian geometry,

fundamental misunderstandings and **entirely unnecessary confusion** concerning IUT continue to persist in certain sectors of the mathematical community. This includes profoundly factually inaccurate assertions to the effect that "the experts agree that the theory is mathematically incorrect". On the other hand, **countless attempts** by the author of IUT, as well as other mathematicians involved with IUT, over the past decade --- cf., e.g., e-mails sent on [June 30, 2022](#), and [December 30, 2022](#), neither of which received any response! --- to pin down

- **precisely who** these "experts" are and/or
- **precisely what** the **mathematical content** is that these "experts" assert to be "mathematically incorrect"

have only been met with **stubborn refusals to respond** to such queries. Here, it is worth noting that this situation constitutes a **very serious violation** of article (6.) of the subsection entitled "Responsibilities of authors" of the Code of Practice of the European Mathematical Society (cf. [EMSCOP](#)):

Mathematicians should not make public claims of potential new theorems or the resolution of particular mathematical problems unless they are able to provide full details in a timely manner.

This confusion has also manifested itself in a sort of **bizarre schizophrenia** on the part of many senior, high-ranking members of the mathematical community. That is to say,

- whereas some senior, high-ranking members of the European mathematical community have maintained for years, both in oral communications to the author of IUT, as well as in numerous internet posts, that the incorrect mathematical content referred to above revolves around the issue of "**redundant copies**" (as discussed in detail in [\[EssLgc\]](#)), other senior, high-ranking members of the European mathematical community have asserted strongly that they could not believe that this issue of "redundant copies" could be the focal point of objections to the mathematical validity of IUT, on the grounds that they could not believe that fellow senior, high-ranking members of the European mathematical community could be confused about such an elementary mathematical issue;
- whereas some senior, high-ranking members of the European mathematical community have maintained for years, both in oral communications to mathematicians involved with IUT and in written form (cf., e.g., the Zentralblatt review of the IUT papers!), that "the experts agree that the theory is mathematically incorrect", one (very) senior, high-ranking member of the European mathematical community has **asserted categorically** (in a personal oral communication) that **neither he nor his colleagues take such assertions** (of a mathematical gap in IUT) **seriously!**

In light of the circumstances just described, one central aspect of dissemination activities concerning IUT over the past decade has consisted of stimulating the worldwide mathematical

community to strongly encourage mathematicians to

- (*) bear **explicit responsibility** for any assertion that denies the essential mathematical validity of IUT by expressing any such assertion --- not in an informal manner, via rough oral communications, internet posts, or nonrigorously formulated arguments, but rather --- in the form of a **precisely formulated mathematical paper** with **complete, rigorous proofs** that is written in such a fashion that it is suitable for publication in a well-established mathematical journal.

This goal (*) may be understood as the goal of achieving compliance with the passage of the [EMSCOP](#) quoted above. Alternatively, this goal (*) may be understood as the goal of addressing, in a fundamental and definitive fashion, the entirely unnecessary confusion and schizophrenia discussed above, by putting assertions of the sort referred to in (*) in a form that is **well-defined** and can hence be subject, via **conventional scholarly activities** (i.e., workshops/seminars, further research, etc.), to the **scrutiny of the mathematical community**, not only in the present, but also for decades (or even centuries!) to come (cf. the discussion of the **historical significance** of explicit written documentation of mathematical arguments in [\[EssLgc\]](#), § 1.5).

Perhaps the **IUT Challenger Prize** announced by Mr. Nobuo Kawakami in July 2023 can further stimulate the worldwide mathematical community to realize the importance, for the entire field of mathematics, of conducting oneself in a fashion consistent with (*).

In some sense, the **model** of the many of the dissemination activities referred to above may be understood as the discussions between Emmanuel Lepage (a maître de conférences at Sorbonne University in France) and the author of IUT during the years 2017 - 2021. At the outset of these discussions (in the summer of 2017), Lepage took a deeply skeptical position with regard to the mathematical validity of IUT. On the other hand, it was precisely as a result of his sincere efforts during the period 2017 - 2021 to respond to repeated requests by the author of IUT to make his objections to the mathematical validity of IUT **precise** and **explicit** that Lepage was finally able to realize and acknowledge explicitly that his objections to IUT were purely psychological, that he had misunderstood IUT, and that he no longer had any mathematical reasons not to acknowledge the mathematical validity of IUT. These discussions with Lepage formed the basis of a substantial portion of the discussion given in the latter portion of [\[EssLgc\]](#), § 3. Thus, in summary, one may think of the goal of the dissemination activities referred to above as the goal of

(**) performing a **massive global search** for professional mathematicians who assert that they have **mathematical reasons** not to acknowledge the mathematical validity of IUT, in the hope that stimulating such mathematicians to perform the "exercise" of documenting these mathematical reasons in a **precise** and **explicit** fashion can trigger a **comparably dramatic evolution of understanding** for such mathematicians to the evolution of understanding experienced by Lepage during discussions with the author of IUT in 2017 - 2021.

Finally, we note that, in this context, it is of fundamental importance to distinguish between

- mathematicians who do not acknowledge the mathematical validity of IUT for **nonmathematical** (such as **social/political/psychological**) reasons and
- mathematicians who do not acknowledge the mathematical validity of IUT for **mathematical** reasons.

Indeed, one way to understand the goal of the dissemination activities referred to above is as the goal of issuing

(***) a **clear and unequivocal message** to the worldwide mathematical community that the denial of the mathematical validity of a mathematical theory published in a well-established mathematical journal for **nonmathematical** reasons is **entirely unacceptable** (cf., e.g., the passage of the [EMSCOP](#) quoted above!), and that the fundamental importance, from the point of view of the sound development of the entire field of pure mathematics, of providing **mathematical** reasons for any such denial of the mathematical validity of a mathematical theory **transcends the specific case of IUT** and indeed cuts to the very **core** of what it means to **practice pure mathematics**.